

A-Level Maths

Pure P24

Vectors

Learning and Revision Booklet

Suitable for students studying Pearson Edexcel A-Level Mathematics
Original explanations, practice questions and worked answers.

Method Marks

Free learning resource

About this booklet

Important note

This resource is independently created. It is not affiliated with, endorsed by, or produced by Pearson Edexcel. All questions, examples, diagrams and worked solutions are original.

This booklet teaches the vector skills that appear in A-Level Pure Mathematics. It is designed to help you understand what a vector represents, use vector notation accurately and solve geometry problems using clear method.

How to use it

Step	What to do
1	Read the key facts before trying the examples.
2	Cover the working and try each example yourself.
3	Complete the guided practice before moving to harder questions.
4	Mark your work using the fully worked answers.
5	Go back to any section where your working was unclear or incomplete.

Contents

Section	Topic
1	Vector notation and operations
2	Magnitude, direction and unit vectors
3	Position vectors and line segments
4	Vector equations of lines
5	Vectors in three dimensions
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7	Challenge questions
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Before you start

You should already be comfortable with coordinates, directed line segments, Pythagoras and solving simple simultaneous equations. Vectors combine these skills with notation.

1. Vector notation and operations

A vector has size and direction. It can describe a movement, a force, a velocity or the position of a point from the origin. In A-Level Maths, vectors are often written using column vectors or using $\mathbf{i}$ and $\mathbf{j}$ notation.

Key facts

A vector is not just a point. It describes a movement from one position to another.

Vectors can be added, subtracted and multiplied by a scalar.

Parallel vectors are scalar multiples of each other.

$$\begin{pmatrix} x \\ y \end{pmatrix} = x\mathbf{i} + y\mathbf{j}$$

$$\vec{AB} = \mathbf{b} - \mathbf{a}$$

Method: working with vectors

Step	What to do
1	Write each vector in the same notation.
2	Add or subtract corresponding components.
3	Multiply every component by the scalar if the vector is being scaled.
4	Use position vectors carefully: from A to B means final position minus starting position.

Example 1: adding and subtracting vectors

Question: If $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + 5\mathbf{j}$, find $2\mathbf{a} - \mathbf{b}$.

Working:

$$2\mathbf{a} = 6\mathbf{i} + 4\mathbf{j}$$

$$2\mathbf{a} - \mathbf{b} = 6\mathbf{i} + 4\mathbf{j} - (-\mathbf{i} + 5\mathbf{j})$$

$$= 7\mathbf{i} - \mathbf{j}$$

Answer: $7\mathbf{i} - \mathbf{j}$

Example 2: finding a vector between two points

Question: A has coordinates (2, -1) and B has coordinates (7, 5). Find vector AB.

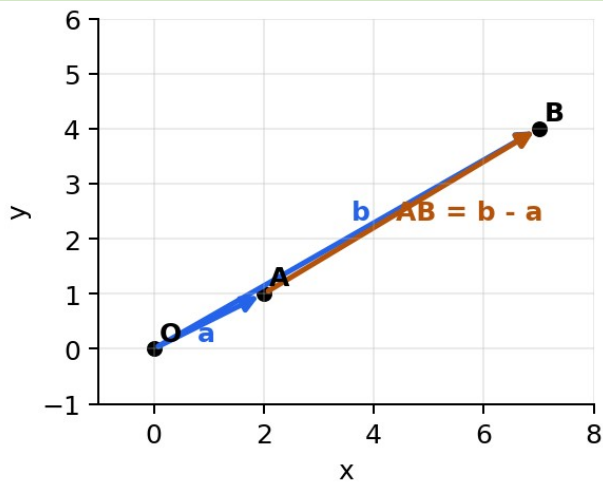
Working:

From A to B means B minus A.

$$\text{Change in } x: 7 - 2 = 5$$

$$\text{Change in } y: 5 - (-1) = 6$$

Answer: $\mathbf{AB} = 5\mathbf{i} + 6\mathbf{j}$



Guided practice 1

Question	Space for working
1. Write the vector $3\mathbf{i} - 2\mathbf{j}$ as a column vector.	
2. Write the column vector $(-4, 5)$ in $\mathbf{i}$ and $\mathbf{j}$ form.	
3. Calculate $(2\mathbf{i} + 3\mathbf{j}) + (5\mathbf{i} - \mathbf{j})$.	
4. Calculate $3(4\mathbf{i} - 2\mathbf{j})$.	
5. Calculate $(6, -1) - (2, 7)$.	
6. Find vector $\mathbf{AB}$ when A is $(2, -1)$ and B is $(7, 3)$.	
7. Find the position vector of A when A has coordinates $(-3, 5)$.	
8. If $\mathbf{a} = (4, -1)$ and $\mathbf{b} = (-2, 6)$, find $2\mathbf{a} - \mathbf{b}$.	

2. Magnitude, direction and unit vectors

The magnitude of a vector is its length. For a 2D vector, use Pythagoras. A unit vector has length 1 and points in the same direction as the original vector.

Key facts

Magnitude means length.

A unit vector keeps the direction but changes the length to 1.

Distance between two points can be found by first finding the vector between them.

$$|\mathbf{a}| = \sqrt{x^2 + y^2}$$

$$\text{unit vector} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Method: finding magnitude and unit vectors

Step	What to do
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1	Identify the horizontal and vertical components.
2	Square the components and add them.
3	Square root the result to find the magnitude.
4	For a unit vector, divide every component by the magnitude.

Example 1: finding the magnitude

Question: Find the magnitude of vector $6\mathbf{i} - 8\mathbf{j}$.

Working:

Use Pythagoras with components 6 and -8.

$$\text{Magnitude} = \sqrt{6^2 + (-8)^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100}$$

$$= 10$$

Answer: 10

Example 2: finding a unit vector

Question: Find a unit vector in the direction of $5\mathbf{i} + 12\mathbf{j}$, giving components as decimals.

Working:

$$\text{Magnitude} = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$$

Divide both components by 13.

Answer: $0.385\mathbf{i} + 0.923\mathbf{j}$ to 3 decimal places

Guided practice 2

Question	Space for working
1. Find the magnitude of $3\mathbf{i} + 4\mathbf{j}$.	
2. Find the magnitude of $-5\mathbf{i} + 12\mathbf{j}$.	
3. Find a unit vector in the direction of $6\mathbf{i} + 8\mathbf{j}$, giving components as decimals.	
4. Find the distance between P(1, 2) and Q(5, 8).	
5. Find vector AB and its magnitude when A is (-1, 4) and B is (2, -2).	
6. A velocity vector is $12\mathbf{i} + 5\mathbf{j}$. Find the speed.	

3. Position vectors and line segments

A position vector tells you where a point is from the origin. Once you know the position vectors of two points, you can find midpoints, points on a line segment and vectors between points.

Key facts

If $\mathbf{a}$ and $\mathbf{b}$ are the position vectors of A and B, then vector AB is $\mathbf{b} - \mathbf{a}$.

The midpoint is halfway between the two position vectors.

A point on AB can be written as $\mathbf{a}$ plus a fraction of the movement from A to B.

$$\vec{AB} = \mathbf{b} - \mathbf{a}$$

$$\text{midpoint} = \frac{\mathbf{a} + \mathbf{b}}{2}$$

$$\mathbf{p} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$$

Method: using position vectors

Step	What to do
1	Write the position vectors of the points clearly.
2	For a vector from one point to another, subtract starting position from final position.
3	For a midpoint, average the coordinates.
4	For a point in a ratio, move the required fraction of the way from the start point to the end point.

Example 1: finding a midpoint

Question: A is (2, 5) and B is (8, 1). Find the midpoint of AB.

Working:

Average the x-coordinates: $(2 + 8) \div 2 = 5$

Average the y-coordinates: $(5 + 1) \div 2 = 3$

Answer: (5, 3)

Example 2: finding a point in a ratio

Question: A is (1, 2) and B is (10, 8). P divides AB in the ratio 2:1 from A to B. Find P.

Working:

The total ratio is 3 parts, so P is two parts out of three from A to B.

Vector AB = (9, 6).

Two parts out of three of AB is (6, 4).

$\mathbf{P} = \mathbf{A} + (6, 4) = (7, 6)$.

Answer: P = (7, 6)

Guided practice 3

Question	Space for working
1. Find the midpoint of A(2, 4) and B(8, 10).	
2. M is the midpoint of AB. A is (-1, 5) and M is (3, 1). Find B.	
3. C divides AB in the ratio 2:1 from A to B. A is (1, 2) and B is (7, 11). Find C.	
4. Find a vector equation of the line through A(1, 3) with direction vector $2\mathbf{i} - \mathbf{j}$.	
5. Decide whether P(7, 0) lies on the line $\mathbf{r} = (1, 3) + \lambda(2, -1)$.	
6. Show that A(1, 2), B(5, 8) and C(9, 14) are collinear.	

4. Vector equations of lines

A vector equation of a line needs a point on the line and a direction vector. The parameter tells you how far to travel along the direction vector.

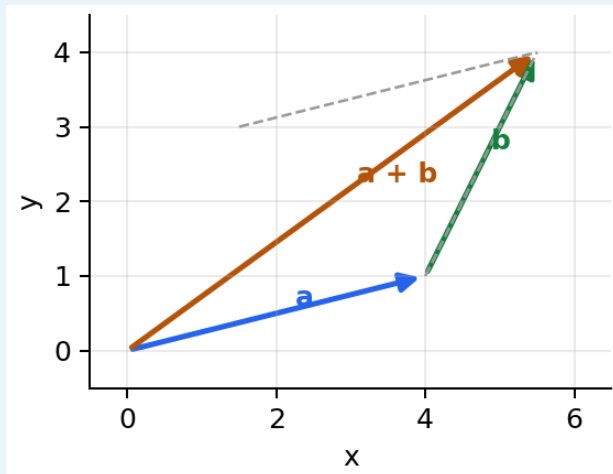
Key facts

In the equation $\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$, $\mathbf{a}$ is a position vector to a point on the line and $\mathbf{d}$ is the direction vector.

Different values of λ give different points on the line.

To test whether a point lies on the line, compare the components and check the same value of λ works.

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$$



Method: checking whether a point is on a line

Step	What to do
1	Substitute the coordinates of the point into the vector equation.
2	Compare one component to find the value of the parameter.
3	Check that the same parameter works in every component.
4	If it works in all components, the point is on the line.

Example 1: writing a vector equation of a line

Question: Write a vector equation of the line through A(3, -2) with direction vector $4\mathbf{i} + \mathbf{j}$.

Working:

Position vector to A is (3, -2).

Direction vector is (4, 1).

Answer: $\mathbf{r} = (3, -2) + \lambda(4, 1)$

Example 2: testing a point on a line

Question: Does P(11, 0) lie on $\mathbf{r} = (3, -2) + \lambda(4, 1)$?

Working:

Compare x-components: $3 + 4\lambda = 11$, so $\lambda = 2$.

Check y-component: $-2 + 2 = 0$.

The same value works, so P is on the line.

Answer: yes

5. Vectors in three dimensions

Three-dimensional vectors work in the same way as two-dimensional vectors, but they have an extra component. You can use the same ideas for magnitude, distance, line equations and collinearity.

Key facts

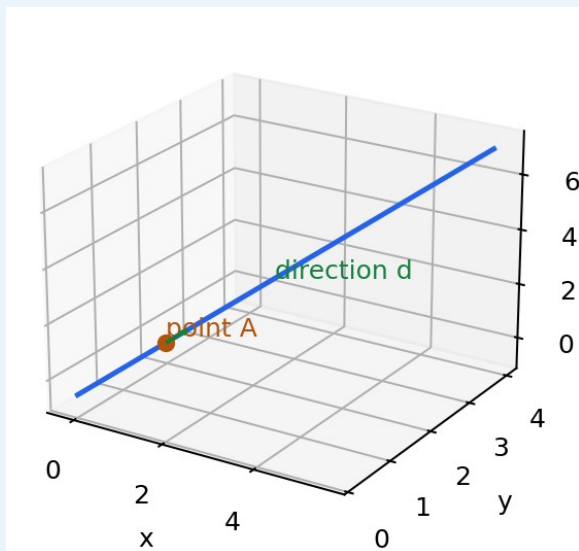
A 3D vector has x, y and z components.

The magnitude formula is just Pythagoras extended into three dimensions.

A 3D line equation still needs a point and a direction vector.

$$|\mathbf{a}| = \sqrt{x^2 + y^2 + z^2}$$

$$\mathbf{r} = (x_0, y_0, z_0) + \lambda(a, b, c)$$



Method: working in 3D

Step	What to do
1	Keep the x, y and z components in the same order.
2	For distance, subtract coordinates to find the vector between points.
3	Use the 3D magnitude formula.
4	For a line, write a position vector plus a parameter times a direction vector.
5	For collinearity, check whether one vector is a scalar multiple of another.

Example 1: distance in 3D

Question: Find the distance between A(1, 2, -1) and B(4, -2, 5).

Working:

Vector AB = (3, -4, 6).

Distance = $\sqrt{3^2 + (-4)^2 + 6^2}$

= $\sqrt{9 + 16 + 36}$

= $\sqrt{61}$

Answer: $\sqrt{61}$

Example 2: checking collinearity in 3D

Question: A(1, 2, 3), B(3, 6, 11) and C(4, 8, 15). Show that the points are collinear.

Working:

Vector $AB = (2, 4, 8)$.

Vector $AC = (3, 6, 12)$.

AC is 1.5 times AB .

The vectors are parallel and share point A , so the points are collinear.

Answer: A , B and C are collinear.

Guided practice 4

Question	Space for working
1. Find the magnitude of the vector $(2, -3, 6)$.	
2. Find vector AB when A is $(1, 2, -1)$ and B is $(4, -2, 5)$.	
3. Find the distance between $A(1, 2, -1)$ and $B(4, -2, 5)$.	
4. Find the midpoint of $A(0, 2, -4)$ and $B(6, -2, 8)$.	
5. Write a vector equation of the line through $A(1, 0, 2)$ with direction vector $(3, -1, 4)$.	
6. Check whether $P(7, -2, 10)$ lies on the line $r = (1, 0, 2) + \lambda(3, -1, 4)$.	
7. Find k if $(k, 2, 6)$ lies on $r = (1, 0, 2) + \lambda(2, 1, 2)$.	
8. Show that $A(0, 1, 2)$, $B(2, 3, 4)$ and $C(5, 7, 8)$ are not collinear.	

6. Mixed exam-style questions

Question	Space for working
1. [2 marks] If $a = 2i - j$ and $b = -3i + 4j$, find $3a - 2b$.	
2. [3 marks] A is $(-2, 3)$ and B is $(4, -5)$. Find vector AB and the length AB .	
3. [3 marks] A is $(1, 2)$ and B is $(9, 14)$. P divides AB in the ratio $3:1$ from A to B . Find P .	
4. [3 marks] The line $r = (2, -1) + \lambda(3, 5)$ passes through a point with y -coordinate 14 . Find the x -coordinate of this point.	
5. [3 marks] Determine whether $A(1, 0, 4)$, $B(4, 6, 10)$ and $C(7, 12, 16)$ are collinear.	
6. [3 marks] Find the distance between $A(2, -1, 5)$ and $B(-2, 3, 8)$.	
7. [3 marks] $A(1, 2)$, $B(6, 4)$ and $D(3, 8)$ are three vertices of parallelogram $ABCD$. Find C .	
8. [4 marks] A is $(2, 3)$ and B is $(10, 7)$. Point P lies on AB and has x -coordinate 6 . Find P and the ratio $AP:PB$.	

7. Challenge questions

Question	Space for working
1. A(1, 4), B(7, 2) and C(3, 10) form a triangle. M is the midpoint of BC. Find vector AM and a vector equation of the median from A.	
2. A(1, 2, 0) and B(5, 6, 4). Point P lies on AB such that AP:PB = 3:1. Find the position vector of P.	
3. A(2, -1), B(8, 2), C(10, 6) and D(4, 3). Use vectors to prove that ABCD is a parallelogram.	
4. A(1, 2, 3), B(4, 8, 15) and C(2, 4, 7). Show that the points are collinear and find the ratio AC:CB.	

8. Common mistakes

Mistake	How to avoid it
Subtracting in the wrong order	Vector AB means B - A, not A - B.
Treating a vector as a point	A point has coordinates. A vector describes a movement or position.
Forgetting the third component in 3D	Always keep x, y and z components in the correct order.
Cancelling direction when finding unit vectors	A unit vector must keep the same direction and have magnitude 1.
Assuming two vectors are parallel by sight	Check whether one vector is a scalar multiple of the other.

9. Fully worked answers

Marking your answers

Check that your final answer is in the correct form and that your working shows the direction of the vector clearly.

Guided practice 1 answers

- (3, -2)
- $-4\mathbf{i} + 5\mathbf{j}$
- $7\mathbf{i} + 2\mathbf{j}$
- $12\mathbf{i} - 6\mathbf{j}$
- (4, -8)
- $\mathbf{AB} = (5, 4)$
- $\mathbf{OA} = (-3, 5)$
- $2\mathbf{a} - \mathbf{b} = (10, -8)$

Guided practice 2 answers

- 5
- 13
- $0.6\mathbf{i} + 0.8\mathbf{j}$
- Vector PQ = (4, 6), so distance = $\sqrt{52} = 2\sqrt{13}$
- $\mathbf{AB} = (3, -6)$, magnitude = $\sqrt{45} = 3\sqrt{5}$

6. Speed = $\sqrt{12^2 + 5^2} = 13$

Guided practice 3 answers

1. (5, 7)
2. B = (7, -3)
3. C = (5, 8)
4. $\mathbf{r} = (1, 3) + \lambda(2, -1)$
5. Yes. $\lambda = 3$ gives (7, 0).
6. AB = (4, 6) and BC = (4, 6), so the points are collinear.

Guided practice 4 answers

1. $\sqrt{49} = 7$
2. AB = (3, -4, 6)
3. Distance = $\sqrt{61}$
4. (3, 0, 2)
5. $\mathbf{r} = (1, 0, 2) + \lambda(3, -1, 4)$
6. Yes. $\lambda = 2$ gives (7, -2, 10).
7. y-component gives $\lambda = 2$, so k = 5.
8. AB = (2, 2, 2) and AC = (5, 6, 6). These are not scalar multiples, so the points are not collinear.

Mixed exam-style answers

1. $3\mathbf{a} = 6\mathbf{i} - 3\mathbf{j}$ and $2\mathbf{b} = -6\mathbf{i} + 8\mathbf{j}$. So $3\mathbf{a} - 2\mathbf{b} = 12\mathbf{i} - 11\mathbf{j}$.
2. AB = (6, -8). Length AB = $\sqrt{6^2 + (-8)^2} = 10$.
3. AB = (8, 12). P is three quarters of the way from A to B, so P = (7, 11).
4. $-1 + 5\lambda = 14$, so $\lambda = 3$. Then $x = 2 + 3(3) = 11$.
5. AB = (3, 6, 6) and BC = (3, 6, 6), so the points are collinear.
6. AB = (-4, 4, 3). Distance = $\sqrt{16 + 16 + 9} = \sqrt{41}$.
7. In a parallelogram, $\mathbf{C} = \mathbf{B} + \mathbf{D} - \mathbf{A} = (6, 4) + (3, 8) - (1, 2) = (8, 10)$.
8. x-coordinate: $2 + 8t = 6$, so $t = 0.5$. Then $y = 3 + 4(0.5) = 5$. P = (6, 5) and AP:PB = 1:1.

Challenge answers

1. M is the midpoint of B(7, 2) and C(3, 10), so M = (5, 6). AM = (4, 2). A vector equation of the median is $\mathbf{r} = (1, 4) + \lambda(4, 2)$.
2. AB = (4, 4, 4). P is three quarters of the way from A to B, so P = (1, 2, 0) + (3, 3, 3) = (4, 5, 3). Position vector OP = (4, 5, 3).
3. AB = (6, 3) and DC = D - C = (-6, -3). These are equal in magnitude and opposite in direction. Also AD = (2, 4) and BC = (2, 4). Opposite sides are parallel and equal, so ABCD is a parallelogram.
4. AB = (3, 6, 12) and AC = (1, 2, 4). Since AB = 3AC, the points are collinear. CB = (2, 4, 8), so AC:CB = 1:2.

10. Revision checklist

Tick	Skill
☐	I can add and subtract vectors.
☐	I can multiply a vector by a scalar.
☐	I can find the vector between two points.
☐	I can find the magnitude of a 2D vector.
☐	I can find a unit vector.
☐	I can find midpoints and points dividing line segments.
☐	I can write vector equations of lines.
☐	I can test whether a point lies on a vector line.

☐	I can work with 3D vectors.
☐	I can use scalar multiples to check collinearity.