

A-Level Maths
Pure P20
Integration Basics

Learning and Revision Booklet

Suitable for students studying Pearson Edexcel A-Level Mathematics

Original explanations, practice questions and worked answers.

Method Marks

Free learning resource

About this booklet

Important note

This resource is independently created. It is not affiliated with, endorsed by, or produced by Pearson Edexcel. All questions, examples, diagrams and worked solutions are original.

This booklet introduces the core integration skills needed in A-Level Pure Mathematics. It focuses on reverse differentiation, finding constants of integration, evaluating definite integrals and using integration to find areas under curves.

How to use it

Step	What to do
1	Read the key facts before trying the worked examples.
2	Cover the working and try each example yourself.
3	Complete the guided practice before moving to harder questions.
4	Mark your work using the fully worked answers.
5	Go back to any section where your working was unclear or incomplete.

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Section	Topic
1	Reverse differentiation
2	Finding the constant of integration
3	Definite integrals
4	Area under a curve
5	Roots, negative area and exact answers
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Before you start

You should already be confident with differentiating powers of x , simplifying expressions, substituting values into functions and using exact values such as fractions and surds. Integration reverses many of the differentiation skills you already know.

1. Reverse differentiation

Integration can be thought of as reverse differentiation. If differentiating lowers the power by 1, integration raises the power by 1.

Key fact

For powers of x , add 1 to the power and then divide by the new power. Always include the constant of integration for an indefinite integral.

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c, \quad n \neq -1$$

Important exception

The power rule shown above does not apply when the power is -1 .

$$\int x^{-1} dx = \ln|x| + c$$

Method: integrating powers of x

Step	What to do
1	Rewrite roots and reciprocal powers as powers of x if needed.
2	Add 1 to the power.
3	Divide by the new power.
4	Multiply by any coefficient already in front of the term.
5	Add $+ c$ for an indefinite integral.

Example 1: basic integration

Question: Integrate $6x^2 - 4x + 7$.

Working:

$$\begin{aligned}
 &\int (6x^2 - 4x + 7) dx \\
 &= \frac{6x^3}{3} - \frac{4x^2}{2} + 7x + c \\
 &= 2x^3 - 2x^2 + 7x + c
 \end{aligned}$$

Guided practice 1

Question	Space for working
1. Integrate $5x^4$.	
2. Integrate $12x^3 - 6x$.	
3. Integrate $8x^7 + 3x^2 - 5$.	
4. Integrate $9x^2 - 4x + 11$.	
5. Integrate $15x^4 - 10x^3 + 2$.	
6. Explain why you must add $+ c$ for an indefinite integral.	

2. Finding the constant of integration

Sometimes you are given a gradient function and one point on the curve. Integrate first, then use the point to find the constant.

Method: finding the original function

Step	What to do
1	Integrate the gradient function.
2	Include $+ c$.
3	Substitute the given x and y values.
4	Solve for c .
5	Write the full equation of the curve.

Example 2: finding the equation of a curve

Question: A curve has gradient function shown below and passes through $(1, 10)$. Find the equation of the curve.

Working:

$$\frac{dy}{dx} = 12x^2 - 6x + 4$$

$$y = 4x^3 - 3x^2 + 4x + c$$

$$10 = 4 - 3 + 4 + c \Rightarrow c = 5$$

$$y = 4x^3 - 3x^2 + 4x + 5$$

Guided practice 2

Question	Space for working
1. A curve has gradient $4x + 3$ and passes through $(2, 10)$. Find its equation.	
2. A curve has gradient $9x^2 - 4$ and passes through $(1, 3)$. Find its equation.	
3. A curve has gradient $6x^2 + 2x$ and passes through $(0, 5)$. Find its equation.	

Question	Space for working
4. A curve has gradient $20x^3 - 6x$ and passes through (1, 8). Find its equation.	
5. A curve has gradient $3x^2 - 12x + 5$ and passes through (2, 1). Find its equation.	
6. Explain why the given point must be substituted into the original curve, not the gradient function.	

3. Definite integrals

A definite integral has limits. Instead of giving a function with + c, it gives a numerical value. The constant cancels when the upper and lower limits are subtracted.

Key fact

If $F(x)$ is an integral of $f(x)$, then the definite integral from a to b is found by substituting the limits.

$$\int_a^b f(x) dx = F(b) - F(a)$$

Method: evaluating a definite integral

Step	What to do
1	Integrate the function.
2	Write the limits clearly.
3	Substitute the upper limit into the integrated expression.
4	Substitute the lower limit into the integrated expression.
5	Subtract the lower result from the upper result.

Example 3: definite integration

Question: Evaluate the definite integral.

Working:

$$\begin{aligned}
 &\int_1^3 (2x + 1) dx \\
 &= [x^2 + x]_1^3 \\
 &= (9 + 3) - (1 + 1) = 10
 \end{aligned}$$

Guided practice 3

Question	Space for working
1. Evaluate the integral of $4x$ from 0 to 3.	
2. Evaluate the integral of $3x^2$ from 1 to 2.	
3. Evaluate the integral of $2x + 5$ from 2 to 6.	

Question	Space for working
4. Evaluate the integral of $x^2 + 2x$ from 0 to 4.	
5. Evaluate the integral of $6x^2 - 4x$ from 1 to 3.	
6. Explain why $+ c$ is not needed in the final answer to a definite integral.	

4. Area under a curve

A definite integral can give the signed area between a curve and the x-axis. When the curve is above the x-axis, this is the area under the curve.

Key fact

When the curve is on or above the x-axis between the limits, the area is found by integrating y with respect to x .

$$\text{Area} = \int_a^b y \, dx \quad \text{when } y \geq 0$$

Method: area under a curve

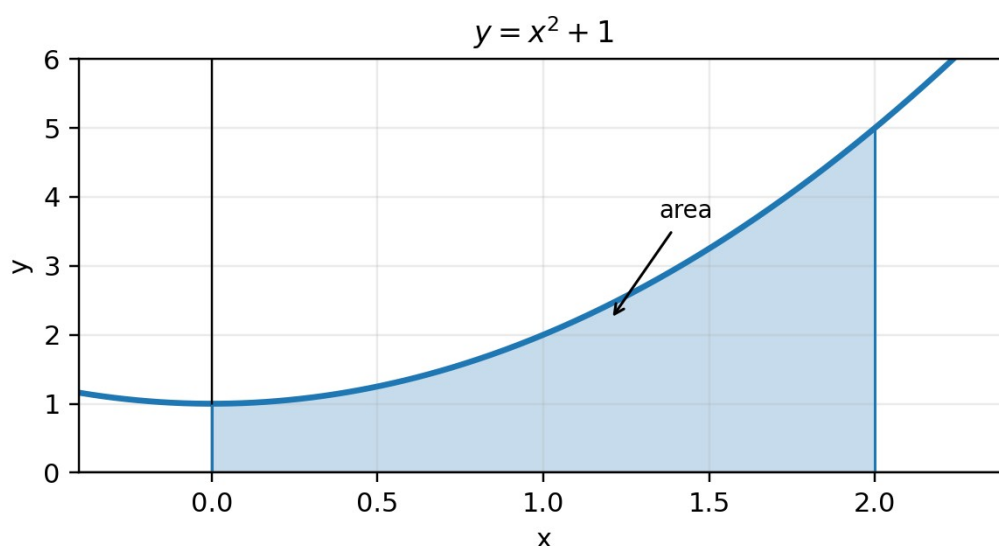
Step	What to do
1	Check the limits of the region.
2	Check whether the curve is above or below the x-axis.
3	Integrate the curve equation.
4	Substitute the upper and lower limits.
5	Give the exact area with square units if needed.

Example 4: area under a positive curve

Question: Find the exact area under $y = x^2 + 1$ from $x = 0$ to $x = 2$.

Working:

$$\begin{aligned}
 & \int_0^2 (x^2 + 1) \, dx \\
 &= \left[\frac{x^3}{3} + x \right]_0^2 \\
 &= \frac{8}{3} + 2 = \frac{14}{3}
 \end{aligned}$$



Guided practice 4

Question	Space for working
1. Find the area under $y = 2x$ from $x = 0$ to $x = 4$.	
2. Find the area under $y = x^2$ from $x = 0$ to $x = 3$.	
3. Find the area under $y = x^2 + 2$ from $x = 1$ to $x = 3$.	
4. Find the area under $y = 3x^2 - 2x + 1$ from $x = 0$ to $x = 2$.	
5. Find the exact area under $y = 4 - x^2$ from $x = 0$ to $x = 2$.	
6. Explain why it is useful to sketch the curve before finding an area.	

5. Roots, negative area and exact answers

If part of a curve is below the x-axis, the definite integral gives a negative signed area. Actual area is always positive, so you may need to split the region at the roots.

Key idea

A definite integral below the x-axis is negative because it is signed area.

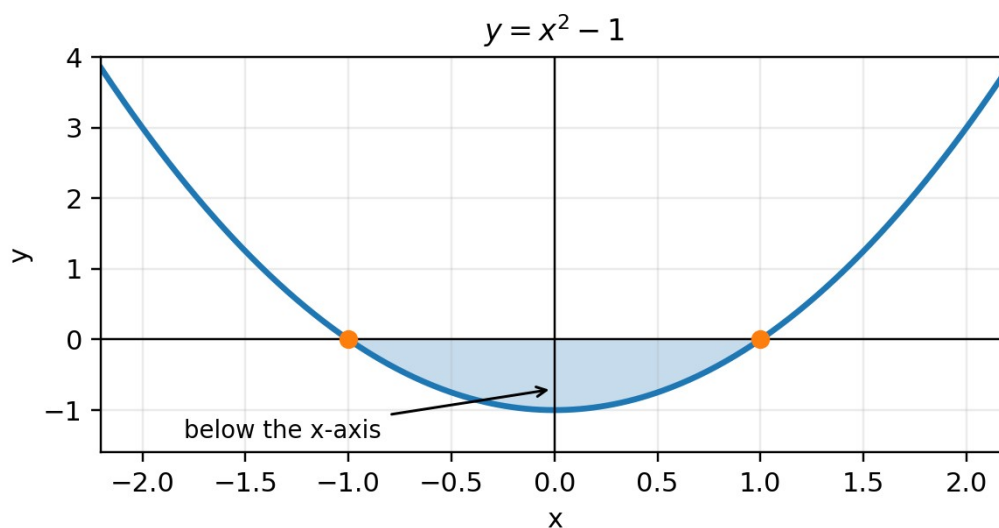
For actual area, split the region at any roots and take positive areas.

Example 5: negative signed area

Question: Evaluate the signed area between $y = x^2 - 1$ and the x-axis from $x = -1$ to $x = 1$.

Working:

$$\int_{-1}^1 (x^2 - 1) dx = -\frac{4}{3}$$



The negative sign tells us the curve is below the x-axis over this interval. The actual area is four thirds square units.

Example 6: roots and fractional powers

Question: Integrate $4\sqrt{x} + 6x^{-2}$.

Working:

$$\begin{aligned} & \int (4\sqrt{x} + 6x^{-2}) dx \\ &= \int (4x^{\frac{1}{2}} + 6x^{-2}) dx \\ &= \frac{8}{3}x^{\frac{3}{2}} - 6x^{-1} + c \\ &= \frac{8}{3}x\sqrt{x} - \frac{6}{x} + c \end{aligned}$$

Guided practice 5

Question	Space for working
1. Evaluate the signed area for $y = x - 2$ from $x = 0$ to $x = 2$.	
2. Find the actual area between $y = x - 2$ and the x -axis from $x = 0$ to $x = 4$.	
3. Integrate $6\sqrt{x}$.	
4. Integrate $10x^{-3}$.	
5. Integrate $3x^2 + 4\sqrt{x}$.	
6. Explain the difference between signed area and actual area.	

6. Mixed exam-style questions

Question	Space for working
1. [3 marks] Integrate $12x^3 - 8x + 5$.	
2. [4 marks] A curve has gradient $6x^2 - 4x$ and passes through $(1, 7)$. Find its equation.	
3. [3 marks] Evaluate the integral of $5x^4$ from 0 to 2.	
4. [4 marks] Find the exact area under $y = x^2 + 3$ from $x = 1$ to $x = 4$.	
5. [4 marks] Evaluate the integral of $4x^3 - 2x$ from 1 to 3.	
6. [5 marks] A curve has gradient $3x^2 + 2x - 6$ and passes through $(0, -4)$. Find the equation of the curve.	
7. [4 marks] Find the area between $y = 6 - 2x$ and the x -axis from $x = 0$ to $x = 3$.	
8. [5 marks] Find the actual area between $y = x^2 - 4$ and the x -axis from $x = -2$ to $x = 2$.	
9. [4 marks] Integrate $8\sqrt{x} - 3x^{-2}$.	
10. [5 marks] The rate of change of a quantity is $12t^2 - 4t$. The quantity is 10 when $t = 0$. Find the quantity when $t = 2$.	

7. Challenge questions

Question	Space for working
C1. A curve has gradient $ax^2 + 6x$ and passes through $(1, 5)$. Its equation contains the term $4x^3$. Find a and the full equation of the curve.	
C2. Find the exact area between $y = x^2 - 2x$ and the x -axis between its roots.	
C3. The definite integral of kx^2 from 0 to 3 is 36. Find k .	
C4. A curve has gradient $4x^3 - 12x$ and passes through $(0, 5)$. Find the coordinates of the points where the curve crosses the x -axis, if any.	
C5. Find the value of a such that the area under $y = ax +$	

Question	Space for working
2 from $x = 0$ to $x = 4$ is 24.	

8. Common mistakes

Mistake	How to fix it
Forgetting + c after an indefinite integral.	Add + c unless the integral has limits or the question asks for a definite value.
Dividing by the old power instead of the new power.	Add 1 to the power first, then divide by the new power.
Using the gradient function when substituting a point.	Substitute the point into the integrated curve equation.
Subtracting the limits in the wrong order.	Use upper limit first, then subtract the lower limit.
Treating signed area as actual area without checking the graph.	If the curve is below the x-axis, the integral is negative. Actual area must be positive.
Leaving roots or fractional powers in an awkward form.	Rewrite roots and reciprocals as powers before integrating.

9. Fully worked answers

Guided practice 1

- $x^5 + c$.
- $3x^4 - 3x^2 + c$.
- $x^8 + x^3 - 5x + c$.
- $3x^3 - 2x^2 + 11x + c$.
- $3x^5 - 2x^4 + 2x + c$.
- Indefinite integration gives a family of curves, so + c represents the unknown vertical shift.

Guided practice 2

- $y = 2x^2 + 3x + c$. Using (2, 10): $10 = 8 + 6 + c$, so $c = -4$. $y = 2x^2 + 3x - 4$.
- $y = 3x^3 - 4x + c$. Using (1, 3): $3 = 3 - 4 + c$, so $c = 4$. $y = 3x^3 - 4x + 4$.
- $y = 2x^3 + x^2 + c$. Using (0, 5): $c = 5$.
- $y = 4x^5 - 3x^2 + c$. Using (1, 8): $8 = 4 - 3 + c$, so $c = 7$.
- $y = x^3 - 6x^2 + 5x + c$. Using (2, 1): $1 = 8 - 24 + 10 + c$, so $c = 7$.
- The point lies on the original curve, not on the gradient curve.

Guided practice 3

- Integral of $4x$ is $2x^2$. From 0 to 3: 18.
- Integral of $3x^2$ is x^3 . From 1 to 2: $8 - 1 = 7$.
- Integral is $x^2 + 5x$. From 2 to 6: $66 - 14 = 52$.
- Integral is one third of x^3 plus x^2 . From 0 to 4: 64 thirds plus 16, which is 112 thirds.
- Integral is $2x^3 - 2x^2$. From 1 to 3: $36 - 0 = 36$.
- Any + c cancels when $F(b) - F(a)$ is calculated.

Guided practice 4

- Area = 16 square units.
- Area = 9 square units.

- Area = 44 thirds square units.
- Area = 6 square units.
- Area = 16 thirds square units.
- A sketch helps show whether the curve is above or below the x-axis and whether the region needs splitting.

Guided practice 5

- The signed area is -2 square units.
- Split at $x = 2$. The actual area is 4 square units.
- Integral is $4x\sqrt{x} + c$.
- Integral is $-5x^{-2} + c$, or negative five over x squared plus c .
- Integral is x^3 plus eight thirds $x\sqrt{x}$ plus c .
- Signed area can be negative below the x-axis. Actual area is a positive measure of size.

Mixed exam-style questions

- $3x^4 - 4x^2 + 5x + c$.
- $y = 2x^3 - 2x^2 + c$. Using (1, 7): $7 = 2 - 2 + c$, so $c = 7$. Therefore $y = 2x^3 - 2x^2 + 7$.
- Integral is x^5 . From 0 to 2: 32.
- Integral is one third of x^3 plus $3x$. From 1 to 4 gives 72 thirds, so the area is 24 square units.
- Integral is $x^4 - x^2$. From 1 to 3 gives 72.
- $y = x^3 + x^2 - 6x + c$. Since $y = -4$ when $x = 0$, $c = -4$.
- Integral of $6 - 2x$ from 0 to 3 gives 9 square units.
- The curve is below the x-axis between -2 and 2. The signed integral is negative 32 thirds, so the actual area is 32 thirds square units.
- Integral is sixteen thirds $x\sqrt{x}$ plus $3x^{-1}$ plus c .
- $Q = 4t^3 - 2t^2 + c$. Since $Q = 10$ when $t = 0$, $c = 10$. When $t = 2$, $Q = 32 - 8 + 10 = 34$.

Challenge questions

- Integrating ax^2 gives one third of ax^3 . If this is $4x^3$, then $a = 12$. $y = 4x^3 + 3x^2 + c$. Using (1, 5): $5 = 4 + 3 + c$, so $c = -2$.
- $y = x^2 - 2x$ has roots 0 and 2 and is below the x-axis between them. The signed integral is negative four thirds, so the actual area is four thirds square units.
- Integral of kx^2 is one third of kx^3 . From 0 to 3 gives $9k$. So $9k = 36$ and $k = 4$.
- $y = x^4 - 6x^2 + 5$. The x-axis crossings solve $x^4 - 6x^2 + 5 = 0$. Let $u = x^2$, so $u^2 - 6u + 5 = 0$. Hence $u = 1$ or 5 , so $x = \pm 1$ or $\pm\sqrt{5}$.
- The area is the integral of $ax + 2$ from 0 to 4, which is $8a + 8$. Set $8a + 8 = 24$, so $a = 2$.

10. Revision checklist

Skill	Tick
I can integrate simple powers of x .	☐
I can remember to add $+ c$ for indefinite integrals.	☐
I can use a point to find the constant of integration.	☐
I can evaluate definite integrals using limits.	☐
I can find the area under a curve when the curve is above the x-axis.	☐
I can recognise when signed area is negative.	☐
I can split a region at roots when finding actual area.	☐
I can write exact answers clearly.	☐