

A-Level Maths

Pure P18

Differentiation Techniques

Learning and Revision Booklet

Suitable for students studying Pearson Edexcel A-Level Mathematics

Original explanations, practice questions and worked answers.

Method Marks

Free learning resource

About this booklet

Important note

This resource is independently created. It is not affiliated with, endorsed by, or produced by Pearson Edexcel. All questions, examples, diagrams and worked solutions are original.

This booklet develops the differentiation techniques used across A-Level Pure Mathematics. It covers the chain rule, product rule, quotient rule, and the main derivatives for trigonometric, exponential and logarithmic functions.

How to use it

Step	What to do
1	Read the key facts before trying the examples.
2	Cover the working and try each example yourself.
3	Complete the guided practice before moving to harder questions.
4	Mark your work using the fully worked answers.
5	Go back to any section where your method choice was unclear.

Contents

Section	Topic
1	Choosing the right technique
2	The chain rule
3	The product rule
4	The quotient rule
5	Trigonometric derivatives
6	Exponential and logarithmic derivatives
7	Implicit differentiation
8	Mixed exam-style questions
9	Challenge questions
10	Common mistakes

11	Fully worked answers
12	Revision checklist

Before you start

You should already be comfortable with differentiating powers of x , finding gradients at points and forming equations of tangents and normals.

1. Choosing the right technique

Most differentiation questions begin with choosing the right method. Some expressions can be differentiated term by term, but others need a special rule.

Key facts

Use the chain rule when one function is inside another function.

Use the product rule when two functions of x are multiplied together.

Use the quotient rule when one function of x is divided by another function of x .

Use implicit differentiation when x and y are mixed together in the same equation.

Method: choosing a differentiation technique

Step	What to do
1	First simplify or expand if that makes the expression easier.
2	Look for brackets with a power: this often suggests the chain rule.
3	Look for two separate functions multiplied together: this suggests the product rule.
4	Look for a fraction with functions of x on top and bottom: this suggests the quotient rule.
5	If the equation contains both x and y , consider implicit differentiation.

Example 1: choosing the method

Question: Choose a suitable method for differentiating each expression.

Working:

$y = (3x^2 - 1)^5$ uses the chain rule.

$y = x^2 \sin x$ uses the product rule.

$y = (x^2 + 1)/(x - 3)$ uses the quotient rule.

Answer: chain rule, product rule, quotient rule.

Guided practice 1

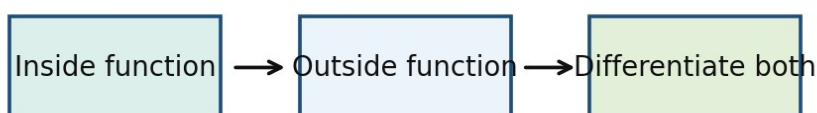
Question	Space for working
1. Choose a method for $y = (2x + 5)^7$.	
2. Choose a method for $y = x^3e^x$.	
3. Choose a method for $y = \sin x/(x^2 + 1)$.	
4. Choose a method for $x^2 + y^2 = 25$.	
5. Choose a method for $y = \ln(4x - 1)$.	

2. The chain rule

The chain rule is used when one function is inside another function. It is often needed for brackets, roots, exponentials, logarithms and trigonometric functions with something more than x inside.

Key fact

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$



Chain rule: derivative of outside $\times$ derivative of inside

Method: using the chain rule

Step	What to do
1	Identify the inside function.
2	Differentiate the outside function, keeping the inside unchanged.
3	Multiply by the derivative of the inside function.
4	Simplify the final answer.

Example 1: bracket power

Question: Differentiate $y = (2x + 3)^5$.

Working:**Differentiate the outside:** $5(2x + 3)^4$.**Differentiate the inside:** 2.**Multiply them together:**

$$\frac{dy}{dx} = 10(2x + 3)^4$$

Answer: gradient function = $10(2x + 3)^4$.**Example 2: chain rule with a quadratic inside****Question:** Differentiate $y = (x^2 - 4x)^3$.**Working:****Differentiate the outside:** $3(x^2 - 4x)^2$.**Differentiate the inside:** $2x - 4$.**Multiply:**

$$\frac{dy}{dx} = 3(x^2 - 4x)^2(2x - 4)$$

Answer: gradient function = $3(x^2 - 4x)^2(2x - 4)$.**Guided practice 2**

Question	Space for working
1. Differentiate $y = (3x - 1)^6$.	
2. Differentiate $y = (x^2 + 5)^4$.	
3. Differentiate $y = (4 - 2x)^5$.	
4. Differentiate $y = \sqrt{2x + 1}$.	
5. Differentiate $y = \ln(5x + 2)$.	
6. Differentiate $y = e^{3x}$.	

3. The product rule

The product rule is used when two functions of x are multiplied together and expanding would be difficult or impossible.

Key fact

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Product rule keeps one factor unchanged while differentiating the other

first $\times$ derivative of second

+

second $\times$ derivative of first

Method: using the product rule

Step	What to do
1	Label the two factors as u and v.
2	Differentiate u and v separately.
3	Use u times the derivative of v, plus v times the derivative of u.
4	Simplify where possible.

Example 1: product with exponential

Question: Differentiate $y = x^2 e^x$.

Working:

Let $u = x^2$ and $v = e^x$.

Then $du/dx = 2x$ and $dv/dx = e^x$.

Use the product rule:

$$\frac{dy}{dx} = x^2 e^x + e^x (2x)$$

Answer: gradient function = $e^x(x^2 + 2x)$.

Example 2: product with trigonometry

Question: Differentiate $y = x \cos x$.

Working:

Let $u = x$ and $v = \cos x$.

Then $du/dx = 1$ and $dv/dx = -\sin x$.

$$\frac{dy}{dx} = x(-\sin x) + (\cos x)(1)$$

Answer: gradient function = $\cos x - x \sin x$.

Guided practice 3

Question	Space for working
1. Differentiate $y = x e^x$.	

Question	Space for working
2. Differentiate $y = x^2 \sin x$.	
3. Differentiate $y = x \ln x$.	
4. Differentiate $y = (x^2 + 1)e^x$.	
5. Differentiate $y = x^3 \cos x$.	
6. Differentiate $y = (2x - 1)\sin x$.	

4. The quotient rule

The quotient rule is used when one function of x is divided by another function of x . Sometimes it is easier to rewrite the expression using negative powers, but the quotient rule is useful when the denominator is more complicated.

Key fact

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Method: using the quotient rule

Step	What to do
1	Label the numerator as u and the denominator as v .
2	Differentiate u and v separately.
3	Use v times derivative of u , minus u times derivative of v , all over v squared.
4	Simplify the numerator if possible.

Example 1: quotient rule

Question: Differentiate $y = x^2/(x + 1)$.

Working:

Let $u = x^2$ and $v = x + 1$.

Then $du/dx = 2x$ and $dv/dx = 1$.

$$\frac{dy}{dx} = \frac{(x+1)(2x) - x^2(1)}{(x+1)^2}$$

$$= \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2}$$

Answer: gradient function = $(x^2 + 2x)/(x+1)^2$.

Example 2: quotient with trigonometry

Question: Differentiate $y = \sin x / x$.

Working:

Let $u = \sin x$ and $v = x$.

Then $du/dx = \cos x$ and $dv/dx = 1$.

$$\frac{dy}{dx} = \frac{x \cos x - \sin x}{x^2}$$

Answer: gradient function = $(x \cos x - \sin x)/x^2$.

Guided practice 4

Question	Space for working
1. Differentiate $y = x/(x+2)$.	
2. Differentiate $y = (x^2 + 1)/(x - 1)$.	
3. Differentiate $y = e^x/x$.	
4. Differentiate $y = \ln x/x^2$.	
5. Differentiate $y = \cos x/(x+1)$.	
6. Differentiate $y = (3x - 1)/(2x + 5)$.	

5. Trigonometric derivatives

For A-Level Maths, you need the standard derivatives of sine, cosine and tangent. These are normally used with the chain rule when the angle is more than x .

Key facts

$$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x \quad \frac{d}{dx}(\tan x) = \sec^2 x$$

Method: differentiating trigonometric functions

Step	What to do
1	Use the standard derivative for sin, cos or tan.
2	If the angle is not just x, multiply by the derivative of the angle using the chain rule.
3	Keep signs carefully, especially when differentiating cos x.
4	Leave the answer in exact trigonometric form unless asked to evaluate it.

Example 1: basic trigonometric derivatives

Question: Differentiate $y = 4 \sin x - 3 \cos x$.

Working:

$4 \sin x$ becomes $4 \cos x$.

$-3 \cos x$ becomes $3 \sin x$.

Answer: gradient function = $4 \cos x + 3 \sin x$.

Example 2: trigonometry with chain rule

Question: Differentiate $y = \sin(3x + 1)$.

Working:

Derivative of $\sin(3x + 1)$ is $\cos(3x + 1)$.

Multiply by the derivative of $3x + 1$, which is 3.

Answer: gradient function = $3\cos(3x + 1)$.

Guided practice 5

Question	Space for working
1. Differentiate $y = \sin x + \cos x$.	
2. Differentiate $y = 5\cos x$.	
3. Differentiate $y = \tan x - 2\sin x$.	
4. Differentiate $y = \sin(2x)$.	
5. Differentiate $y = \cos(4x - 1)$.	
6. Differentiate $y = \tan(3x)$.	

6. Exponential and logarithmic derivatives

Exponential and logarithmic functions appear often in modelling, calculus and differential equations.
The natural exponential e^x differentiates to itself.

Key facts

$$\frac{d}{dx}(e^x) = e^x \quad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

Method: differentiating exponentials and logarithms

Step	What to do
1	Use the standard derivative for e^x or $\ln x$.
2	If the input is more than x , use the chain rule.
3	For logarithms, check that the argument is positive where needed.
4	Simplify using index or logarithm laws only when it makes the answer clearer.

Example 1: exponential with chain rule

Question: Differentiate $y = e^{2x}$.

Working:

Derivative of e^{2x} is e^{2x} multiplied by the derivative of $2x$.

Answer: gradient function = $2e^{2x}$.

Example 2: logarithm with chain rule

Question: Differentiate $y = \ln(3x - 4)$.

Working:

$$\frac{d}{dx} \ln(3x - 4) = \frac{1}{3x - 4} \times 3$$

Answer: gradient function = $3/(3x - 4)$.

Guided practice 6

Question	Space for working
1. Differentiate $y = e^{5x}$.	
2. Differentiate $y = 4e^x - 2\ln x$.	
3. Differentiate $y = \ln(2x + 1)$.	
4. Differentiate $y = e^{x^2}$.	

Question	Space for working
5. Differentiate $y = x \ln x$.	
6. Differentiate $y = e^x \sin x$.	

7. Implicit differentiation

Implicit differentiation is used when x and y are mixed together in an equation, rather than y being written on its own. When differentiating a y term with respect to x , multiply by dy/dx .

Key fact

$$\frac{d}{dx}(y^n) = ny^{n-1} \frac{dy}{dx}$$

Method: implicit differentiation

Step	What to do
1	Differentiate each term with respect to x .
2	Whenever you differentiate a y term, multiply by dy/dx .
3	Collect the dy/dx terms on one side where needed.
4	Factor out dy/dx and rearrange to make dy/dx the subject.

Example 1: circle equation

Question: Find dy/dx for $x^2 + y^2 = 25$.

Working:

Differentiate each term:

$$2x + 2y \frac{dy}{dx} = 0$$

Rearrange:

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Answer: $dy/dx = -x/y$.

Example 2: mixed xy term

Question: Find $\frac{dy}{dx}$ for $xy + y^2 = 6$.

Working:

Differentiate xy using the product rule:

$$\frac{d}{dx}(xy) = x \frac{dy}{dx} + y$$

Differentiate y^2 :

$$2y \frac{dy}{dx}$$

So:

$$x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$$

$$(x + 2y) \frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = -\frac{y}{x + 2y}$$

Answer: $\frac{dy}{dx} = -y/(x + 2y)$.

Guided practice 7

Question	Space for working
1. Find $\frac{dy}{dx}$ for $x^2 + y^2 = 10$.	
2. Find $\frac{dy}{dx}$ for $y^3 + x = 4$.	
3. Find $\frac{dy}{dx}$ for $xy = 12$.	
4. Find $\frac{dy}{dx}$ for $x^2y + y = 5$.	
5. Find $\frac{dy}{dx}$ for $x^2 + xy + y^2 = 7$.	
6. Find the gradient of $x^2 + y^2 = 25$ at the point (3, 4).	

8. Mixed exam-style questions

Question	Space for working
1. [4 marks] Differentiate $y = (4x - 1)^6$.	
2. [4 marks] Differentiate $y = x^2e^x$.	
3. [5 marks] Differentiate $y = (x^2 + 1)/(x - 2)$.	
4. [4 marks] Differentiate $y = \sin(5x) + 3\cos x$.	
5. [4 marks] Differentiate $y = \ln(2x - 3)$.	
6. [5 marks] Differentiate $y = x^3\ln x$.	
7. [5 marks] Find dy/dx for $x^2 + 3y^2 = 18$.	
8. [6 marks] Find the equation of the tangent to $y = e^{2x}$ at $x = 0$.	

9. Challenge questions

Question	Space for working
1. Differentiate $y = (x^2 + 1)^3 \sin x$.	
2. Differentiate $y = \ln(x^2 + 1)/(x + 1)$.	
3. Find dy/dx for $x^2y + xy^2 = 6$, then find the gradient at $(1, 2)$.	
4. The curve $y = x^2e^x$ has a stationary point. Find its x -coordinate and explain your method.	
5. Show that the derivative of $y = \tan(2x)$ can be written as $2\sec^2(2x)$.	

10. Common mistakes

Mistake	How to avoid it
Using the product rule when the chain rule is needed.	Look for one function inside another, such as

	brackets raised to a power.
Forgetting to multiply by the derivative of the inside function.	After differentiating the outside, always multiply by the inside derivative.
Writing the quotient rule in the wrong order.	Use denominator times derivative of numerator, minus numerator times derivative of denominator.
Forgetting the negative sign when differentiating $\cos x$.	Remember that the derivative of $\cos x$ is $-\sin x$.
Differentiating $\ln(kx + a)$ as $1/(kx + a)$ only.	Multiply by the derivative of the inside, so $\ln(kx + a)$ gives $k/(kx + a)$.
Forgetting dy/dx when differentiating y terms implicitly.	Every y term needs a factor of dy/dx because y depends on x .

11. Fully worked answers

Guided practice 1

1. Chain rule.
2. Product rule.
3. Quotient rule.
4. Implicit differentiation.
5. Chain rule with logarithms.

Guided practice 2

1. $18(3x - 1)^5$
2. $8x(x^2 + 5)^3$
3. $-10(4 - 2x)^4$
4. $1/\sqrt{2x + 1}$
5. $5/(5x + 2)$
6. $3e^{3x}$

Guided practice 3

1. $e^x + xe^x = e^x(1 + x)$
2. $2x \sin x + x^2 \cos x$
3. $\ln x + 1$
4. $2xe^x + (x^2 + 1)e^x = e^x(x^2 + 2x + 1)$
5. $3x^2 \cos x - x^3 \sin x$
6. $2 \sin x + (2x - 1) \cos x$

Guided practice 4

1. $2/(x + 2)^2$
2. $[(x - 1)(2x) - (x^2 + 1)]/(x - 1)^2 = (x^2 - 2x - 1)/(x - 1)^2$

3. $(xe^x - e^x)/x^2 = e^x(x - 1)/x^2$

4. $(x^2(1/x) - 2x \ln x)/x^4 = (1 - 2\ln x)/x^3$

5. $[-(x + 1)\sin x - \cos x]/(x + 1)^2$

6. $17/(2x + 5)^2$

Guided practice 5

1. $\cos x - \sin x$

2. $-5\sin x$

3. $\sec^2 x - 2\cos x$

4. $2\cos(2x)$

5. $-4\sin(4x - 1)$

6. $3\sec^2(3x)$

Guided practice 6

1. $5e^{5x}$

2. $4e^x - 2/x$

3. $2/(2x + 1)$

4. $2xe^{x^2}$

5. $\ln x + 1$

6. $e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$

Guided practice 7

1. $dy/dx = -x/y$

2. $dy/dx = -1/(3y^2)$

3. $dy/dx = -y/x$

4. $dy/dx = -2xy/(x^2 + 1)$

5. $dy/dx = -(2x + y)/(x + 2y)$

6. $dy/dx = -x/y$, so at (3, 4) the gradient is $-3/4$.

Mixed exam-style questions

1. $24(4x - 1)^5$

2. $e^x(x^2 + 2x)$

3. $[(x - 2)(2x) - (x^2 + 1)]/(x - 2)^2 = (x^2 - 4x - 1)/(x - 2)^2$

4. $5\cos(5x) - 3\sin x$

5. $2/(2x - 3)$

6. $3x^2 \ln x + x^2$

7. $2x + 6y$ $dy/dx = 0$, so $dy/dx = -x/(3y)$.

8. $y = e^{2x}$, $dy/dx = 2e^{2x}$. At $x = 0$, $y = 1$ and gradient = 2, so tangent is $y = 2x + 1$.

Challenge questions

1. Product rule: derivative = $6x(x^2 + 1)^2 \sin x + (x^2 + 1)^3 \cos x$.

2. Quotient rule: derivative = $[(x + 1)(2x/(x^2 + 1)) - \ln(x^2 + 1)]/(x + 1)^2$.

3. Differentiate: $2xy + x^2 dy/dx + y^2 + 2xy dy/dx = 0$. Therefore $dy/dx = -(2xy + y^2)/(x^2 + 2xy)$. At (1, 2), gradient = $-8/5$.

4. $dy/dx = e^x(x^2 + 2x) = e^x x(x + 2)$. Since e^x is never zero, stationary points occur when $x = 0$ or $x = -2$.

5. $\tan(2x)$ is an outside tan function with inside $2x$. The derivative is $\sec^2(2x) \times 2 = 2\sec^2(2x)$.

12. Revision checklist

Can you do this?	Tick when confident
I can choose whether to use the chain, product or quotient rule.	☐
I can use the chain rule for brackets, logs, exponentials and trigonometric functions.	☐
I can use the product rule for two functions multiplied together.	☐
I can use the quotient rule for one function divided by another.	☐
I know the standard derivatives of $\sin x$, $\cos x$ and $\tan x$.	☐
I know the standard derivatives of e^x and $\ln x$.	☐
I can differentiate implicitly when x and y are mixed together.	☐
I can use differentiation techniques in exam-style problems.	☐