

A-Level Maths

Pure P17

Differentiation Basics

Learning and Revision Booklet

Suitable for students studying Pearson Edexcel A-Level Mathematics

Original explanations, practice questions and worked answers.

Method Marks

Free learning resource

About this booklet

Important note

This resource is independently created. It is not affiliated with, endorsed by, or produced by Pearson Edexcel. All questions, examples, diagrams and worked solutions are original.

This booklet introduces the first differentiation skills needed for A-Level Pure Mathematics. It focuses on understanding the gradient function, differentiating powers of x , and using derivatives to find gradients, tangents and normals.

How to use it

Step	What to do
1	Read the key facts before trying the examples.
2	Cover the working and try each example yourself.
3	Complete the guided practice before moving to harder questions.
4	Mark your work using the fully worked answers.
5	Go back to any section where your working was unclear or incomplete.

Contents

Section	Topic
1	What differentiation means
2	The power rule
3	Constants, multiples and sums
4	Negative and fractional powers
5	Gradients at a point
6	Tangents and normals
7	Mixed exam-style questions
8	Challenge questions
9	Common mistakes

10	Fully worked answers
11	Revision checklist

Before you start

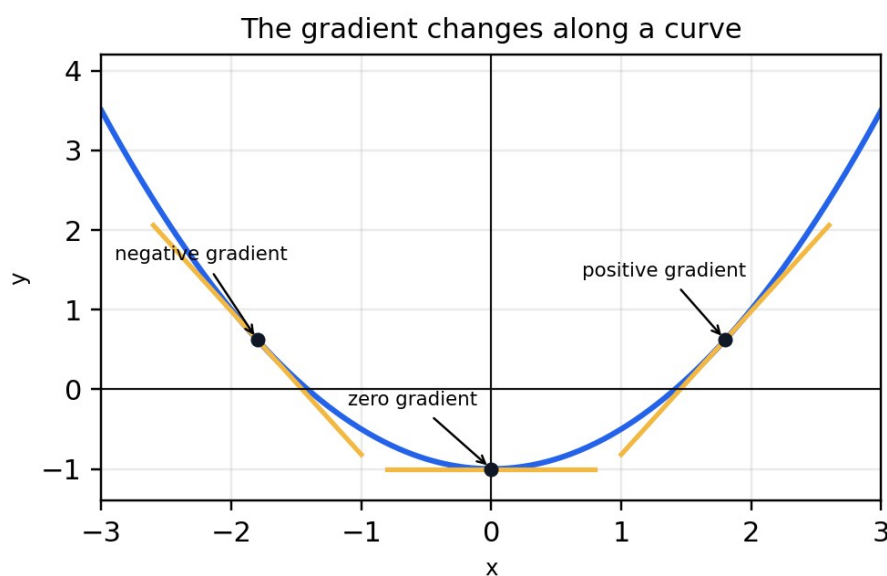
You should already be comfortable with expanding brackets, simplifying powers, solving linear equations and finding the equation of a straight line.

1. What differentiation means

Differentiation is used to find the gradient of a curve. For a straight line the gradient is constant, but for a curve the gradient usually changes from point to point.

Key idea

The derivative gives the gradient function. You can then substitute an x-value to find the gradient at a particular point.

**Key notation**

If y is written in terms of x , the derivative can be written as the gradient function. In exam working, it is usually shown using derivative notation.

$$\frac{dy}{dx}$$

Method: using a derivative

Step	What to do
1	Differentiate the expression to find the gradient function.

Step	What to do
2	Substitute the required x-value into the gradient function.
3	Interpret the result as the gradient at that point.
4	Use the gradient with coordinate geometry if a tangent or normal is needed.

Example 1: interpreting a derivative

Question: The gradient function of a curve is $3x^2 - 4$. Find the gradient when $x = 2$.

Working:

Substitute $x = 2$ into the gradient function:

$$3(2)^2 - 4 = 12 - 4 = 8$$

Answer: the gradient is 8.

Guided practice 1

Question	Space for working
1. A curve has gradient function $4x + 1$. Find the gradient when $x = 3$.	
2. A curve has gradient function $2x^2 - 5$. Find the gradient when $x = -1$.	
3. A curve has gradient function $6x - 7$. Find the gradient when $x = 2$.	
4. A curve has gradient function $x^2 + 3x$. Find the gradient when $x = 4$.	

2. The power rule

The power rule is the main rule used when differentiating powers of x . It is quick, but it only works when the expression is written as a power of x .

Key fact

To differentiate x to the power n , multiply by the power and then reduce the power by 1.

$$y = x^n \quad \Rightarrow \quad \frac{dy}{dx} = nx^{n-1}$$

Method: differentiating powers of x

Step	What to do
1	Identify the power of x .
2	Bring the power down to the front as a multiplier.
3	Reduce the power by 1.
4	Simplify the result.

Example 1: basic power rule**Question:** Differentiate $y = x^5$.**Working:****Bring the power 5 down to the front.****Reduce the power by 1: $5 - 1 = 4$.****Answer: gradient function = $5x^4$.****Example 2: a coefficient with the power rule****Question:** Differentiate $y = 7x^4$.**Working:****The coefficient 7 stays as a multiplier.**

$$7 \times 4x^3 = 28x^3$$

Answer: gradient function = $28x^3$.**Guided practice 2**

Question	Space for working
1. Differentiate $y = x^3$.	
2. Differentiate $y = x^6$.	
3. Differentiate $y = 4x^5$.	
4. Differentiate $y = -3x^4$.	
5. Differentiate $y = 8x^2$.	
6. Differentiate $y = 12x^7$.	

3. Constants, multiples and sums

Differentiation works term by term. Constants disappear because their gradient is zero.

Key facts

The derivative of a constant is 0.

A multiplier stays as a multiplier.

Differentiate each term separately, then collect the result.

Method: differentiating several terms

Step	What to do
1	Differentiate each power of x separately.
2	Remove any constant term, because it differentiates to 0.
3	Keep signs carefully, especially negative terms.
4	Simplify the final gradient function.

Example 1: several terms

Question: Differentiate $y = 3x^4 - 5x^2 + 7x - 9$.

Working:

$3x^4$ becomes $12x^3$.

$-5x^2$ becomes $-10x$.

$7x$ becomes 7 .

-9 becomes 0 .

Answer: gradient function = $12x^3 - 10x + 7$.

Example 2: simplifying before differentiating

Question: Differentiate $y = 2x(3x^2 - 4x + 5)$.

Working:

Expand first:

$y = 6x^3 - 8x^2 + 10x$

Differentiate term by term:

Answer: gradient function = $18x^2 - 16x + 10$.

Guided practice 3

Question	Space for working
1. Differentiate $y = x^4 + 3x^2 - 6$.	
2. Differentiate $y = 5x^3 - 2x^2 + 8x$.	
3. Differentiate $y = -4x^5 + 7x - 1$.	
4. Differentiate $y = 6x^2 - 9x + 11$.	
5. Differentiate $y = 3x(2x^2 + x - 4)$.	

Question	Space for working
6. Differentiate $y = (x + 2)(x^2 - 3x + 1)$.	

4. Negative and fractional powers

Some expressions need to be rewritten before they can be differentiated with the power rule. This is especially important for roots and reciprocal powers.

Key facts

Square roots can be rewritten as powers of one half.

Reciprocals can be rewritten using negative powers.

Once the expression is written as a power of x , the same power rule applies.

$$\sqrt{x} = x^{\frac{1}{2}} \quad \frac{1}{x} = x^{-1} \quad \frac{1}{x^2} = x^{-2}$$

Method: roots and reciprocals

Step	What to do
1	Rewrite each root or reciprocal as a power of x .
2	Use the power rule as normal.
3	Leave the answer as powers of x unless the question asks for a different form.
4	Check that powers have been reduced by 1, including negative and fractional powers.

Example 1: square root

Question: Differentiate $y = \sqrt{x}$.

Working:

$$y = x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

Answer: gradient function $= \frac{1}{2}x^{-1/2}$.

Example 2: reciprocal power

Question: Differentiate $y = 5/x^2$.

Working:

Rewrite as $y = 5x^{-2}$.

Differentiate using the power rule:

$$5 \times -2x^{-3} = -10x^{-3}$$

Answer: gradient function = $-10x^{-3}$.

Guided practice 4

Question	Space for working
1. Rewrite $\sqrt{x}$ as a power of x , then differentiate it.	
2. Differentiate $y = x^{-3}$.	
3. Differentiate $y = 4x^{-2}$.	
4. Differentiate $y = 6\sqrt{x}$.	
5. Differentiate $y = 3/x$.	
6. Differentiate $y = 2x^3 + 5/x^2$.	

5. Gradients at a point

Once you have found the gradient function, substitute the x -coordinate of the point. This gives the gradient of the curve at that point.

Method: finding a gradient at a point

Step	What to do
1	Differentiate to find the gradient function.
2	Substitute the x -coordinate of the point.
3	Calculate the gradient carefully.
4	If needed, find the y -coordinate by substituting into the original equation.

Example 1: gradient at a given x -value

Question: For $y = x^3 - 4x + 2$, find the gradient when $x = 2$.**Working:****Differentiate: gradient function = $3x^2 - 4$.****Substitute $x = 2$:**

$$3(2)^2 - 4 = 12 - 4 = 8$$

Answer: the gradient is 8.

Example 2: gradient at a point on the curve

Question: Find the gradient of $y = 2x^2 - 3x + 1$ at the point where $x = -1$.

Working:

Differentiate: gradient function = $4x - 3$.

Substitute $x = -1$:

$$4(-1) - 3 = -7$$

Answer: the gradient is -7 .

Guided practice 5

Question	Space for working
1. For $y = x^2 + 5x$, find the gradient when $x = 3$.	
2. For $y = x^3 - 2x$, find the gradient when $x = -1$.	
3. For $y = 4x^2 - x + 6$, find the gradient when $x = 2$.	
4. For $y = 2x^3 + 3x^2 - 5$, find the gradient when $x = 1$.	
5. For $y = x^4 - 3x^2 + 2x$, find the gradient when $x = -2$.	

6. Tangents and normals

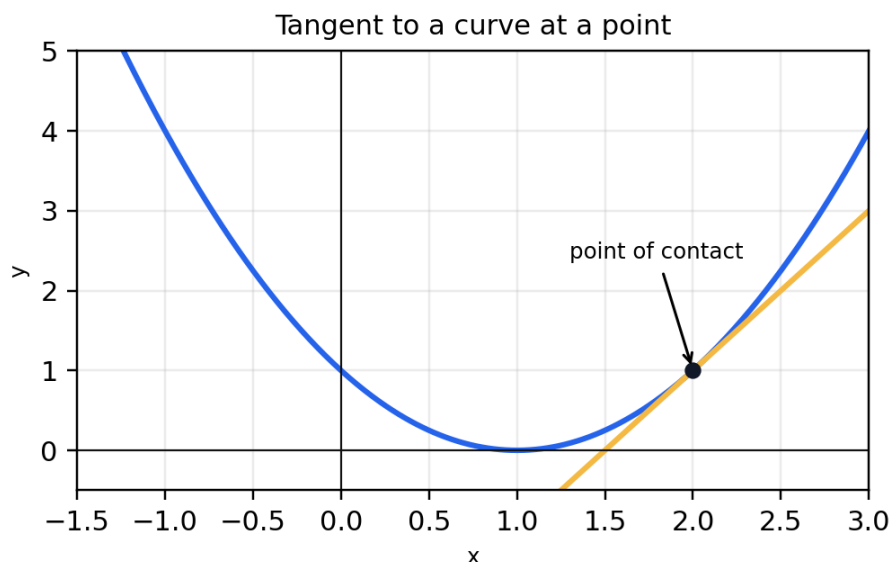
A tangent touches a curve at a point and has the same gradient as the curve at that point. A normal is perpendicular to the tangent.

Key facts

Tangent gradient = gradient of the curve at the point.

Normal gradient is the negative reciprocal of the tangent gradient.

Use $y - y_1 = m(x - x_1)$ to form the equation of the line.



Method: finding a tangent

Step	What to do
1	Differentiate the curve to find the gradient function.
2	Substitute the x-coordinate to find the tangent gradient.
3	Find the y-coordinate if it has not been given.
4	Use the point and gradient in the equation of a straight line.

Example 1: equation of a tangent

Question: Find the equation of the tangent to $y = x^2 + 3x$ at $x = 2$.

Working:

Differentiate: gradient function = $2x + 3$.

At $x = 2$, gradient = $2(2) + 3 = 7$.

Find y : $y = 2^2 + 3(2) = 10$.

Use point (2, 10) and gradient 7:

$$y - 10 = 7(x - 2)$$

$$y = 7x - 4$$

Answer: $y = 7x - 4$.

Example 2: equation of a normal

Question: Find the equation of the normal to $y = x^2 - 4x$ at $x = 3$.

Working:

Differentiate: gradient function = $2x - 4$.

At $x = 3$, tangent gradient = 2.

Normal gradient = $-1/2$.

Find y : $y = 3^2 - 4(3) = -3$.

Use point (3, -3) and gradient $-\frac{1}{2}$:

$$y + 3 = -\frac{1}{2}(x - 3)$$

Answer: $y = -\frac{1}{2}x - 1\frac{1}{2}$.

Guided practice 6

Question	Space for working
1. Find the equation of the tangent to $y = x^2 + 2x$ at $x = 1$.	
2. Find the equation of the tangent to $y = x^3 - x$ at $x = 2$.	
3. Find the equation of the normal to $y = x^2 + 4x$ at $x = 1$.	
4. Find the equation of the tangent to $y = 2x^2 - 3x + 1$ at $x = -1$.	
5. Find the equation of the normal to $y = x^3 + 2x$ at $x = 1$.	

7. Mixed exam-style questions

Question	Space for working
1. [3 marks] Differentiate $y = 5x^4 - 3x^2 + 8x - 7$.	
2. [3 marks] Differentiate $y = 2x(x^2 - 4x + 1)$.	
3. [3 marks] Differentiate $y = 6\sqrt{x} + 4x^3$.	
4. [4 marks] For $y = x^3 - 6x^2 + 2$, find the gradient when $x = 4$.	
5. [4 marks] Find the equation of the tangent to $y = x^2 - 5x$ at $x = 3$.	
6. [5 marks] Find the equation of the normal to $y = 2x^2 + x - 1$ at $x = 1$.	

Question	Space for working
7. [4 marks] The curve $y = ax^3 + bx$ has gradient 14 when $x = 2$. Given $a = 1$, find b .	
8. [5 marks] A curve has equation $y = x^3 - 3x^2 + 4$. Find the points where the gradient is zero.	

8. Challenge questions

Question	Space for working
1. A curve has equation $y = x^3 + ax^2 + bx$. Its gradient is 5 when $x = 1$ and 20 when $x = 2$. Find a and b .	
2. Find the value of k so that the tangent to $y = x^2 + kx$ at $x = 3$ is parallel to the line $y = 10x - 4$.	
3. The normal to $y = x^2 - 2x$ at $x = a$ has gradient $-\frac{1}{4}$. Find a .	
4. A curve has equation $y = x^3 - 6x^2 + 9x$. Find the x -values where the tangent is horizontal and decide whether the gradient changes from positive to negative or negative to positive.	

9. Common mistakes

Mistake	How to avoid it
Forgetting that constants differentiate to 0.	Remember that a constant has no x term, so its gradient is zero.
Reducing the power before multiplying by it.	Bring the power down first, then reduce the power by 1.
Changing signs incorrectly with negative powers.	Write one clear line of working for each negative power.
Using the original equation instead of the derivative to find a gradient.	Differentiate first, then substitute the x -value into the gradient function.
Using the tangent gradient for the normal.	For the normal, use the negative reciprocal of the

tangent gradient.

10. Fully worked answers

Guided practice 1

- 13
- 3
- 5
- 28

Guided practice 2

- $3x^2$
- $6x^5$
- $20x^4$
- $-12x^3$
- $16x$
- $84x^6$

Guided practice 3

- $4x^3 + 6x$
- $15x^2 - 4x + 8$
- $-20x^4 + 7$
- $12x - 9$
- $18x^2 + 6x - 12$
- Expand first: $x^3 - x^2 - 5x + 2$, so gradient function = $3x^2 - 2x - 5$.

Guided practice 4

- $x^{1/2}$ becomes $\frac{1}{2}x^{-1/2}$.
- $-3x^{-4}$
- $-8x^{-3}$
- $3x^{-1/2}$
- $-3x^{-2}$
- $6x^2 - 10x^{-3}$

Guided practice 5

- Gradient function = $2x + 5$, so at $x = 3$ the gradient is 11.
- Gradient function = $3x^2 - 2$, so at $x = -1$ the gradient is 1.
- Gradient function = $8x - 1$, so at $x = 2$ the gradient is 15.
- Gradient function = $6x^2 + 6x$, so at $x = 1$ the gradient is 12.
- Gradient function = $4x^3 - 6x + 2$, so at $x = -2$ the gradient is -18.

Guided practice 6

- Tangent: gradient = 4, point (1, 3), so $y = 4x - 1$.
- Tangent: gradient = 11, point (2, 6), so $y = 11x - 16$.
- Tangent gradient = 6, normal gradient = $-1/6$, point (1, 5), so $y - 5 = -1/6(x - 1)$.
- Tangent: gradient = -7, point (-1, 6), so $y = -7x - 1$.

5. Tangent gradient = 5, normal gradient = $-1/5$, point (1, 3), so $y - 3 = -1/5(x - 1)$.

Mixed exam-style questions

- Gradient function = $20x^3 - 6x + 8$.
- Expand first: $y = 2x^3 - 8x^2 + 2x$, so gradient function = $6x^2 - 16x + 2$.
- Gradient function = $3x^{-1/2} + 12x^2$.
- Gradient function = $3x^2 - 12x$. At $x = 4$, gradient = $48 - 48 = 0$.
- Gradient function = $2x - 5$. At $x = 3$, gradient = 1 and $y = -6$, so tangent is $y = x - 9$.
- Gradient function = $4x + 1$. At $x = 1$, tangent gradient = 5, normal gradient = $-1/5$, point (1, 2), so $y - 2 = -1/5(x - 1)$.
- Gradient function = $3x^2 + b$. At $x = 2$, $12 + b = 14$, so $b = 2$.
- Gradient function = $3x^2 - 6x = 3x(x - 2)$, so gradient is zero when $x = 0$ or $x = 2$. Points are (0, 4) and (2, 0).

Challenge questions

- Gradient function = $3x^2 + 2ax + b$. Using $x = 1$ gives $3 + 2a + b = 5$. Using $x = 2$ gives $12 + 4a + b = 20$. Solving gives $a = 3$ and $b = -4$.
- Gradient function = $2x + k$. At $x = 3$, gradient = $6 + k$. Parallel to $y = 10x - 4$ means gradient 10, so $k = 4$.
- Gradient function = $2x - 2$. Normal gradient is the negative reciprocal. If the normal gradient is $-1/4$, the tangent gradient is 4. So $2a - 2 = 4$ and $a = 3$.
- Gradient function = $3x^2 - 12x + 9 = 3(x - 1)(x - 3)$. Horizontal tangents occur at $x = 1$ and $x = 3$. Test intervals show the gradient changes positive to negative at $x = 1$, then negative to positive at $x = 3$.

11. Revision checklist

Can you do this?	Tick when confident
I can explain that differentiation gives the gradient function.	☐
I can differentiate powers of x using the power rule.	☐
I can differentiate constants, multiples and sums.	☐
I can rewrite roots and reciprocals as powers of x .	☐
I can find the gradient of a curve at a given x -value.	☐
I can find equations of tangents.	☐
I can find equations of normals.	☐
I can identify and fix common differentiation mistakes.	☐