

A-Level Maths
Pure P16
Exponential Modelling
Learning and Revision Booklet

Suitable for students studying Pearson Edexcel A-Level Mathematics
Original explanations, practice questions and worked answers.

Method Marks

Free learning resource

About this booklet

Important note

This resource is independently created. It is not affiliated with, endorsed by, or produced by Pearson Edexcel. All questions, examples, diagrams and worked solutions are original.

This booklet teaches the exponential modelling skills that appear in A-Level Pure Mathematics. It is designed to help you set up models, interpret constants, solve for time and comment sensibly on limitations.

How to use it

Step	What to do
1	Read the key facts before trying the examples.
2	Cover the working and try each example yourself.
3	Complete the guided practice before moving to harder questions.
4	Mark your work using the fully worked answers.
5	Go back to any section where your working was unclear or incomplete.

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2	Growth models
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Before you start

You should already be comfortable with exponentials, logarithms, natural logarithms and rearranging equations. This booklet applies those skills to modelling questions.

1. Understanding exponential models

An exponential model describes a quantity that changes by a fixed proportion over equal time intervals. This makes it useful for repeated growth and repeated decay.

Key facts

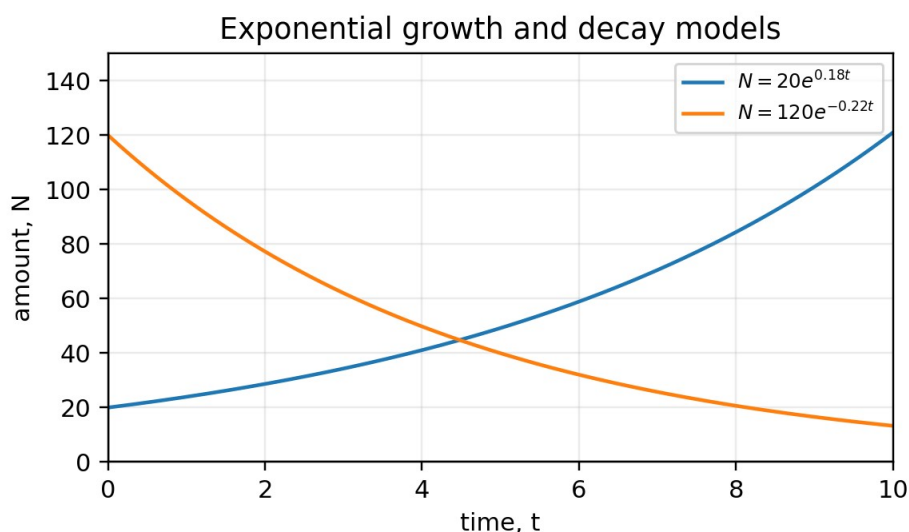
$$N = N_0 e^{kt}$$

N is the amount after time t .

N_0 is the initial amount, when $t = 0$.

k is the rate constant.

If $k > 0$, the model shows growth. If $k < 0$, the model shows decay.

**Method: interpreting an exponential model**

Step	What to do
1	Identify the initial value by putting $t = 0$.
2	Look at the sign of the rate constant to decide whether the model shows growth or decay.
3	Use the model to calculate values by substitution.
4	Use logarithms when you need to solve for time.

Example 1: interpreting the constants

Question: A quantity is modelled by

$$N = 50e^{0.12t}$$

where t is measured in hours. State the initial amount and whether the model shows growth or decay.

Working:

When $t = 0$, $e^{0.12 \times 0} = e^0 = 1$.

So $N = 50$ at the start.

The rate constant is positive, so the model shows growth.

Answer: initial amount = 50; growth model.

Example 2: substituting into a model

Question: A population is modelled by

$$P = 800e^{0.04t}$$

where t is measured in years. Estimate the population after 6 years.

Working:

$$P = 800e^{0.04(6)} = 800e^{0.24}$$

$P = 1017.0$ to one decimal place.

Answer: about 1017 people.

Guided practice 1

Question	Space for working
1. For this model, state the initial amount: $N = 30e^{0.2t}$	
2. For this model, state whether it is growth or decay: $N = 12e^{-0.4t}$	
3. Calculate N when $t = 5$: $N = 20e^{0.1t}$	
4. Calculate N when $t = 3$: $N = 100e^{-0.2t}$	
5. A model has $k = 0.08$. Does it show growth or decay?	
6. A model has $k = -0.15$. Does it show growth or decay?	
7. Find the initial value of: $A = 250e^{-0.03t}$	
8. Explain what the 900 means in: $P = 900e^{0.05t}$	

2. Growth models

In a growth model, the rate constant is positive. The quantity increases slowly at first and then more quickly as time increases.

Key facts

$$N = N_0 e^{kt} \quad \text{with} \quad k > 0$$

A larger value of k means faster growth.

Exponential growth is not the same as adding a fixed amount each time.

Method: forming a growth model

Step	What to do
1	Use the starting value as N_0 .
2	Substitute a known value of N and t into the model.
3	Solve for k using natural logarithms.
4	Write the full model clearly and use it to answer the question.

Example 1: finding the rate constant

Question: A population starts at 400. After 5 years it is 620. Model the population by

$$P = 400e^{kt}$$

Find k to 3 significant figures.

Working:

Substitute $P = 620$ and $t = 5$:

$$620 = 400e^{5k}$$

$$\frac{620}{400} = e^{5k}$$

$$\frac{31}{20} = e^{5k}$$

Take natural logarithms:

$$5k = \ln\left(\frac{31}{20}\right)$$

$$k = \frac{\ln\left(\frac{31}{20}\right)}{5} = 0.0877 \text{ to 3 s.f.}$$

Answer: $k = 0.0877$ to 3 significant figures.

Example 2: using the model to predict

Question: A bacteria culture is modelled by

$$B = 120e^{0.35t}$$

where t is measured in hours. Estimate B after 4 hours.

Working:

$$B = 120e^{0.35(4)} = 120e^{1.4}$$

$B = 486.6$ to one decimal place.

Answer: about 487 bacteria.

Guided practice 2

Question	Space for working
1. For this model, calculate P after 4 years: $P = 200e^{0.06t}$	
2. For this model, calculate B after 3 hours: $B = 50e^{0.4t}$	
3. A quantity starts at 80 and grows to 100 after 2 years. Use $N = 80e^{kt}$ to find k .	
4. A population starts at 500 and grows to 700 after 6 years. Find k .	
5. A model has $N = 40e^{0.18t}$. Find the time when $N = 100$.	
6. A model has $P = 300e^{0.05t}$. Find P after 10 years.	
7. A model has $B = 25e^{0.7t}$. Find the time when $B = 200$.	
8. Explain why exponential growth may become unrealistic over a long time.	

3. Decay models

In a decay model, the rate constant is negative. The quantity decreases by a fixed proportion over equal time intervals.

Key facts

$$N = N_0e^{-kt} \quad \text{with} \quad k > 0$$

This is equivalent to $N = N_0e^{ct}$ where c is negative.

The quantity gets closer to 0 but does not become negative in this model.

Method: using a decay model

Step	What to do
1	Check that the exponent is negative for decay.

Step	What to do
2	Substitute values carefully, including the negative sign.
3	When solving for time, divide by the starting amount first.
4	Take natural logarithms and remember that $\ln$ of a number between 0 and 1 is negative.

Example 1: finding the remaining amount**Question:** A chemical has mass

$$M = 80e^{-0.15t}$$

where t is measured in hours. Find the mass after 6 hours.**Working:**

$$M = 80e^{-0.15(6)} = 80e^{-0.9}$$

 $M = 32.5$ to one decimal place.**Answer:** 32.5 g to one decimal place.**Example 2: finding the time in a decay model****Question:** A substance is modelled by

$$A = 200e^{-0.08t}$$

Find the time when $A = 120$.**Working:**

$$120 = 200e^{-0.08t}$$

$$\frac{120}{200} = e^{-0.08t}$$

$$0.6 = e^{-0.08t}$$

Take natural logarithms:

$$\ln(0.6) = -0.08t$$

$$t = \frac{\ln(0.6)}{-0.08} = 6.39 \text{ to 3 s.f.}$$

Answer: 6.39 hours to 3 significant figures.

Guided practice 3

Question	Space for working
1. Find A after 5 hours: $A = 60e^{-0.12t}$	
2. Find M after 8 days: $M = 300e^{-0.03t}$	
3. A substance starts at 100 g and is 70 g after 4 hours. Use $M = 100e^{-kt}$ to find k .	

Question	Space for working
4. A quantity starts at 500 and falls to 350 after 10 days. Find k in $N = 500e^{-kt}$.	
5. In $A = 90e^{-0.2t}$, find the time when $A = 30$.	
6. In $M = 250e^{-0.06t}$, find the time when $M = 100$.	
7. Explain why a decay model approaches 0 but does not become negative.	
8. Give one real-world limitation of an exponential decay model.	

4. Doubling time and half-life

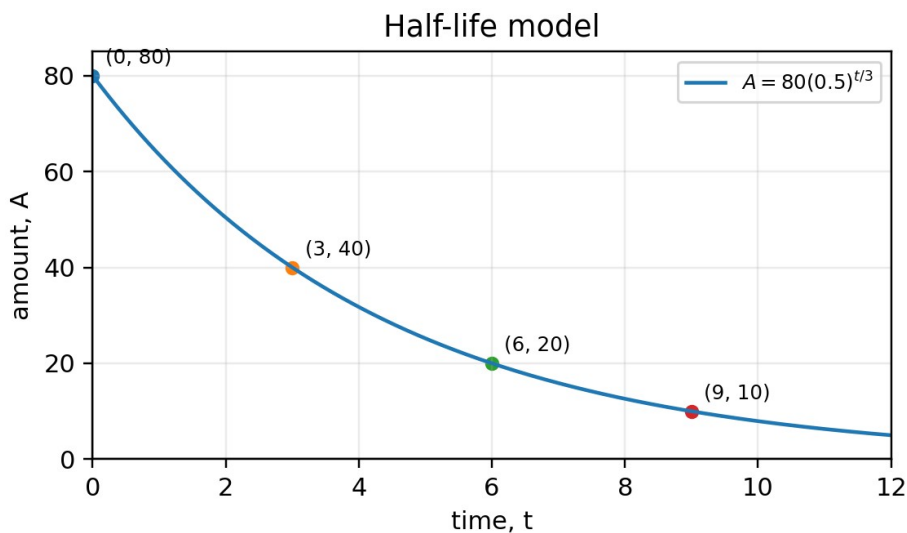
Doubling time is the time taken for a growing quantity to double. Half-life is the time taken for a decaying quantity to halve.

Key facts

$$\text{Doubling time} = \frac{\ln(2)}{k} \quad \text{for } N = N_0e^{kt}$$

$$\text{Half-life} = \frac{\ln(2)}{k} \quad \text{for } N = N_0e^{-kt}$$

The formula looks the same because k is taken as a positive constant in both cases.



Method: finding doubling time or half-life

Step	What to do
1	Write down whether the model is growth or decay.
2	Identify the positive rate constant k .
3	Use $\ln(2)$ divided by k .

Step	What to do
4	Include units in your final answer.

Example 1: doubling time**Question:** A population is modelled by

$$P = 150e^{0.07t}$$

where t is measured in years. Find the doubling time.**Working:**

$$T = \frac{\ln(2)}{0.07} = 9.90$$

Answer: 9.90 years to 3 significant figures.**Example 2: half-life****Question:** A decaying substance is modelled by

$$A = 60e^{-0.18t}$$

where t is measured in days. Find the half-life.**Working:**

$$T = \frac{\ln(2)}{0.18} = 3.85$$

Answer: 3.85 days to 3 significant figures.**Guided practice 4**

Question	Space for working
1. Find the doubling time for $N = 40e^{0.2t}$.	
2. Find the doubling time for $P = 600e^{0.05t}$.	
3. Find the half-life for $A = 90e^{-0.1t}$.	
4. Find the half-life for $M = 500e^{-0.25t}$.	
5. A model has doubling time 12 years. Find k in $N = N_0e^{kt}$.	
6. A model has half-life 8 days. Find k in $A = A_0e^{-kt}$.	
7. Explain why the initial amount does not affect the doubling time.	
8. Explain why the initial amount does not affect the half-life.	

5. Using logarithms to solve modelling problems

Most exponential modelling questions eventually ask for a time. To solve for time, isolate the exponential term and then use natural logarithms.

Method: solving for time in a model

Step	What to do
1	Substitute the target amount into the model.
2	Divide by the initial amount or coefficient first.
3	Take natural logarithms of both sides.
4	Divide by the rate constant and give the answer with units.
5	Check whether the answer is sensible in context.

Example 1: reaching a target amount

Question: The number of users of an app is modelled by

$$U = 2500e^{0.32t}$$

where t is measured in years. Find when U first reaches 10 000.

Working:

$$10000 = 2500e^{0.32t}$$

$$4 = e^{0.32t}$$

$$\ln(4) = 0.32t$$

$$t = \frac{\ln(4)}{0.32} = 4.33$$

Answer: after 4.33 years to 3 significant figures.

Example 2: percentage remaining

Question: A medicine in the body is modelled by

$$M = 50e^{-0.24t}$$

where t is measured in hours. Find when 20% of the medicine remains.

Working:

20% of the original amount is 10 mg.

$$10 = 50e^{-0.24t}$$

$$0.2 = e^{-0.24t}$$

$$\ln(0.2) = -0.24t$$

$$t = \frac{\ln(0.2)}{-0.24} = 6.71$$

Answer: after 6.71 hours to 3 significant figures.

Example 3: commenting on a model

Question: A population is modelled by $P = 1200e^{0.09t}$. Give one reason why the model may not be reliable for large t .

Working:

As t gets large, the model continues to grow without limit. Real populations may be limited by space, food, disease or other factors.

Answer: the model may overestimate the population in the long term because real resources are limited.

Guided practice 5

Question	Space for working
1. In $N = 100e^{0.15t}$, find when $N = 250$.	
2. In $P = 800e^{0.04t}$, find when $P = 1000$.	
3. In $A = 120e^{-0.2t}$, find when $A = 50$.	
4. In $M = 75e^{-0.18t}$, find when $M = 15$.	
5. A quantity grows from 40 to 70 in 5 years. Form a model $N = 40e^{kt}$.	
6. A quantity decays from 200 to 120 in 8 days. Form a model $A = 200e^{-kt}$.	
7. State one limitation of using $N = 50e^{0.3t}$ for a real population.	
8. State one limitation of using $M = 100e^{-0.2t}$ for a real physical process.	

6. Mixed exam-style questions

These questions mix the skills from the booklet. Work neatly and show each step. The marks are a guide to the amount of working expected.

Exam-style questions

Question	Space for working
1. [3 marks] A quantity is modelled by $N = 70e^{0.18t}$. State the initial amount and calculate N when $t = 4$.	
2. [4 marks] A population starts at 600 and reaches 900 after 5 years. Use $P = 600e^{kt}$ to find k .	
3. [4 marks] A chemical has mass $M = 85e^{-0.12t}$. Find the mass after 10 hours.	
4. [4 marks] For $A = 160e^{-0.08t}$, find the time when $A = 100$.	

Question	Space for working
5. [4 marks] Find the doubling time for $P = 250e^{0.06t}$.	
6. [4 marks] Find the half-life for $M = 90e^{-0.14t}$.	
7. [5 marks] A culture grows from 200 to 500 in 6 hours. Form a model $C = 200e^{kt}$, then estimate C after 10 hours.	
8. [5 marks] A medicine has amount $M = 40e^{-0.3t}$. Find the time when 15% of the medicine remains.	

7. Challenge questions

Challenge questions

Question	Space for working
1. A quantity is modelled by $N = ae^{kt}$. It is 20 when $t = 0$ and 45 when $t = 6$. Find a and k .	
2. A decaying substance follows $A = A_0e^{-kt}$. It has half-life 5 hours. Find the percentage remaining after 12 hours.	
3. Two populations are modelled by $P = 300e^{0.08t}$ and $Q = 500e^{0.03t}$. Find when $P = Q$.	
4. A quantity doubles every 9 years. Write a model in the form $N = N_0e^{kt}$.	
5. A substance loses 35% of its mass every 4 days. Find a continuous exponential model for its mass.	

8. Common mistakes

Mistake	How to avoid it
Confusing growth and decay	Check the sign of the exponent. A positive rate constant gives growth. A negative rate constant gives decay.
Forgetting to divide first	Before taking logarithms, isolate the exponential term by dividing by the initial amount or coefficient.
Dropping the negative sign	In decay models, the exponent is negative. Keep the negative sign when solving for time.
Rounding too early	Keep exact logarithmic values during working and only round the final answer.

Mistake	How to avoid it
Ignoring context	Always include units and check whether the answer makes sense in the real situation.

9. Fully worked answers

Marking note

The answers below show one clear method. There may be other valid methods, especially when rearranging logarithmic equations.

Guided practice 1

1. Initial amount = 30.
2. Decay.
3. $N = 20e^{0.5} = 33.0$ to 3 significant figures.
4. $N = 100e^{-0.6} = 54.9$ to 3 significant figures.
5. Growth.
6. Decay.
7. Initial value = 250.
8. 900 is the initial value.

Guided practice 2

1. $P = 200e^{0.24} = 254$ to 3 significant figures.
2. $B = 50e^{1.2} = 166$ to 3 significant figures.
3. $k = \frac{\ln\left(\frac{100}{80}\right)}{2} = 0.112$
4. $k = \frac{\ln\left(\frac{700}{500}\right)}{6} = 0.0561$
5. $t = \frac{\ln\left(\frac{100}{40}\right)}{0.18} = 5.09$
6. $P = 300e^{0.5} = 495$ to 3 significant figures.
7. $t = \frac{\ln(8)}{0.7} = 2.97$
8. It may become unrealistic because real growth is limited by resources.

Guided practice 3

1. $A = 60e^{-0.6} = 32.9$ to 3 significant figures.
2. $M = 300e^{-0.24} = 236$ to 3 significant figures.
3. $k = \frac{-\ln(0.7)}{4} = 0.0892$
4. $k = \frac{-\ln(0.7)}{10} = 0.0357$
5. $t = \frac{\ln\left(\frac{1}{3}\right)}{-0.2} = 5.49$
6. $t = \frac{\ln(0.4)}{-0.06} = 15.3$
7. e^{-kt} is always positive, so the model approaches 0 but does not go below 0.
8. A real process may not decay at a constant proportional rate forever.

Guided practice 4

1. $T = \frac{\ln(2)}{0.2} = 3.47$

$$2. T = \frac{\ln(2)}{0.05} = 13.9$$

$$3. T = \frac{\ln(2)}{0.1} = 6.93$$

$$4. T = \frac{\ln(2)}{0.25} = 2.77$$

$$5. k = \frac{\ln(2)}{12} = 0.0578$$

$$6. k = \frac{\ln(2)}{8} = 0.0866$$

7. The doubling time depends on k , not the starting amount.

8. The half-life depends on k , not the starting amount.

Guided practice 5

$$1. t = \frac{\ln(2.5)}{0.15} = 6.11$$

$$2. t = \frac{\ln(1.25)}{0.04} = 5.58$$

$$3. t = \frac{\ln\left(\frac{50}{120}\right)}{-0.2} = 4.38$$

$$4. t = \frac{\ln(0.2)}{-0.18} = 8.94$$

$$5. k = \frac{\ln\left(\frac{70}{40}\right)}{5} = 0.112$$

$$6. k = \frac{-\ln(0.6)}{8} = 0.0639$$

7. It assumes unlimited growth, which is not realistic for a population.

8. The decay rate may change or reach a background level rather than continuing forever.

Exam-style answers

1.

Initial amount = 70.

$$N = 70e^{0.18 \times 4} = 70e^{0.72} = 144 \text{ to 3 significant figures.}$$

2.

$$900 = 600e^{5k}, \text{ so } 1.5 = e^{5k}.$$

$$k = \frac{\ln(1.5)}{5} = 0.0811$$

3.

$$M = 85e^{-1.2} = 25.6 \text{ to 3 significant figures.}$$

4.

$$100 = 160e^{-0.08t}, \text{ so } 0.625 = e^{-0.08t}.$$

$$t = \frac{\ln(0.625)}{-0.08} = 5.88$$

5.

$$T = \frac{\ln(2)}{0.06} = 11.6$$

6.

$$T = \frac{\ln(2)}{0.14} = 4.95$$

7.

$$500 = 200e^{6k} \Rightarrow k = \frac{\ln(2.5)}{6} = 0.153$$

$$C = 200e^{0.153t}.$$

At $t = 10$, $C = 200e^{1.53} = 923 \text{ to 3 significant figures.}$

8.

15% remains means $M = 6$.

$$6 = 40e^{-0.3t}, \text{ so } 0.15 = e^{-0.3t}.$$

$$t = \frac{\ln(0.15)}{-0.3} = 6.32$$

Challenge answers

1.

At $t = 0$, $a = 20$.

$$45 = 20e^{6k}, \text{ so } 2.25 = e^{6k}.$$

$$k = \frac{\ln(2.25)}{6} = 0.135$$

2.

$$k = \frac{\ln(2)}{5}$$

$$\frac{A}{A_0} = e^{-kt} = e^{-\frac{12\ln(2)}{5}} = 0.189$$

About 18.9% remains.

3.

$$\text{Set } 300e^{0.08t} = 500e^{0.03t}.$$

$$e^{0.05t} = 5 \text{ over } 3.$$

$$t = \frac{\ln(\frac{5}{3})}{0.05} = 10.2$$

4.

$$k = \frac{\ln(2)}{9}$$

$$N = N_0 e^{\frac{\ln(2)}{9}t}$$

5.

After 4 days, 65% remains.

$$0.65 = e^{-4k}.$$

$$k = \frac{-\ln(0.65)}{4} = 0.1077$$

$$M = M_0 e^{-0.1077t}$$

10. Revision checklist

Can I do this?	Check
I can identify the initial value in an exponential model.	☐
I can decide whether a model shows growth or decay.	☐
I can substitute values into an exponential model.	☐
I can form a growth model from given information.	☐
I can form a decay model from given information.	☐
I can solve exponential models for time using logarithms.	☐
I can find doubling time and half-life.	☐
I can comment sensibly on the limitations of an exponential model.	☐